A course on Linear algebra

Ripan Saha

Lecture 1

September 25, 2020

Reference Books

1. Kenneth M Hoffman, Ray Kunze; Linear algebra, *Pearson; 2nd Edition.*

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2. Friedberg, Insel, Spence; Linear Algebra, *Pearson, 5th Edition.*

The big picture of the course

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We shall see that Part A and Part B are actually the two sides of the same coin.

Linear algebra?? What is that??

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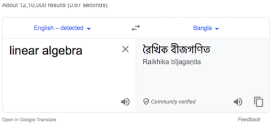
algebra beng.png 

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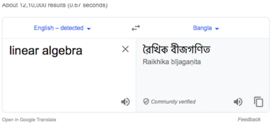
algebra beng.png 

A linear equation is an equation of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

where $a_1, \ldots, a_n, b \in \mathbb{R}$ and $x_1, \ldots, x_n$ are variables. The scalars $a_1, a_2, \ldots, a_n$ are called the coefficients, and b is called the constant term of the equation

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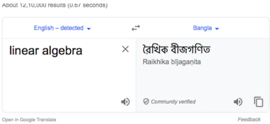
algebra beng.png 

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A linear equation (of n-variables) is called a homogenous equation if $b = 0$ in the above equation.

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i) If $a \neq 0$ then $x = -\frac{b}{a}$.

ii) If $a = 0$, then there are two cases, either $b = 0$ and any x is a solution (infinite number) or $b \neq 0$ and there is no solution (inconsistent equation).

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i) If $b \neq 0$, we can rewrite the equation in the following form

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Thus, for every value of x there is a definite value of y. Thus, in this case there are infinite solutions.

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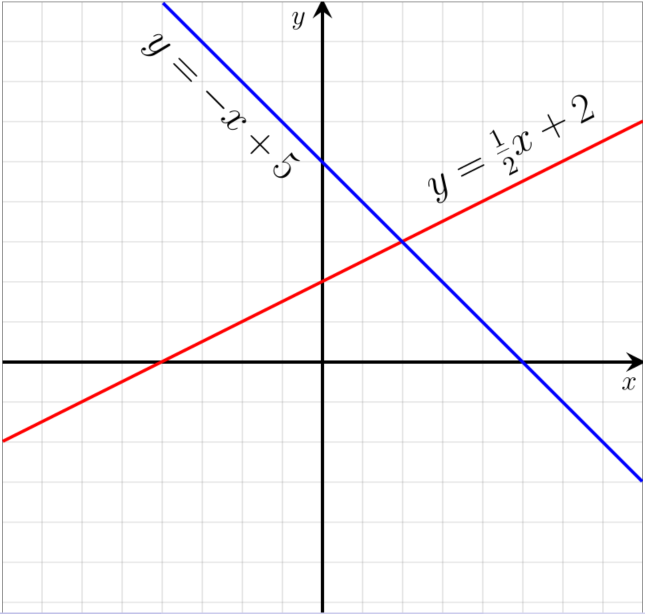
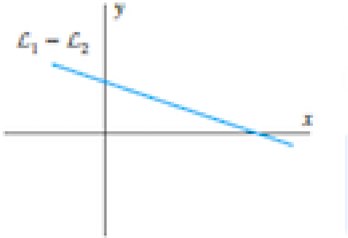
Therefore, for two variables linear equation, there are possibly infinite, no or unique solution.

Solving two linear equations of two variables (Class X)



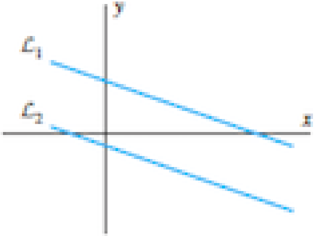
Solving two linear equations of two variables : Infinite solution case



L_1 and L_2 are the same.
Infinitely many solutions

solution.png

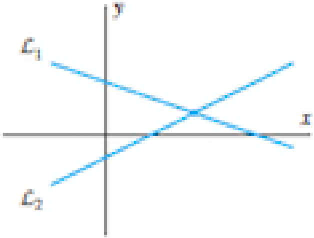
Solving two linear equations of two variables : No solution case



L_1 and L_2 are parallel.
No solution

solution.png

Solving two linear equations of two variables : Infinite solution case



$\mathcal{L}_1$ and $\mathcal{L}_2$ are different but not parallel.
Exactly one solution

solution.png

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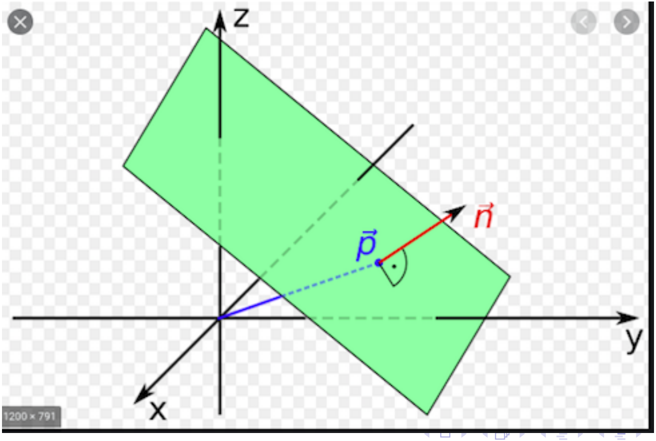
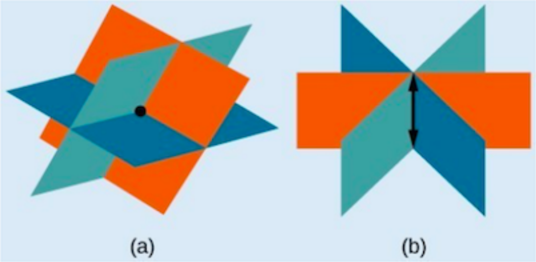
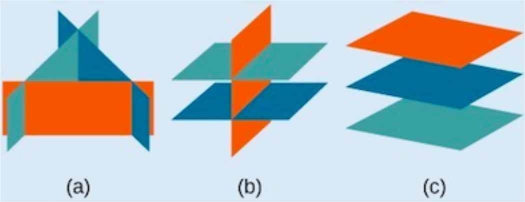
of a plane.png 

Solutions for three variables linear equation



(a)Three planes intersect at a single point, representing a three-by-three system with a single solution. (b) Three planes intersect in a line, representing a three-by-three system with infinite solutions.



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A system of linear equations

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$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m. \end{aligned}$$

where a_{ij} denotes the coefficient of x_j in equation i .

Solution of a system of linear equations

A solution of a system of linear equations in the variables $x_1, x_2, \ldots, x_n$ is a vector $s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$ in $\mathbb{R}^n$ such that every equation in the system is satisfied when each x_i is replaced by s_i.