Functional Forms of Regression Models

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M.Phil/Ph.D Coursework

Beta Coefficients

- "standardized coefficient" or "beta coefficient"
- Idea is to replace *y* and each *x* variable with a standardized version – i.e. subtract mean and divide by standard deviation
- Coefficient reflects standard deviation of *y* for a one standard deviation change in *x*

Functional Form

- OLS can be used for relationships that are not strictly linear in *x* and *y* by using nonlinear functions of *x* and *y* – will still be linear in the parameters

- the natural log of *x, y* or both
- quadratic forms of *x*
- interactions of *x* variables

Interpretation of Log Models

- Double-log model:
- If the model is $\ln(y) = \beta_0 + \beta_1 \ln(x) + u$
- β_1 is the elasticity of *y* with respect to *x*

Interpretation of Log Models

- Semi-log models:

(1) If the model is $\ln(y) = \beta_0 + \beta_1 x + u$

- β_1 is approximately the percentage change in *y* given a 1 unit change in *x*
- If x = time, β_1 = growth rate

Interpretation of Log Models

- Semi-log models:

(2) If the model is $y = \beta_0 + \beta_1 \ln(x) + u$

- β_1 is approximately the change in *y* for a 100 percent change in *x*
- Used in Engel expenditure models:
- German statistician, Ernst Engel (1821-1896):
- “Total expenditure on food tends to increase in AP as total expenditure increases in GP”

Why use log models?

- They give a direct estimate of elasticity
- For models with $y > 0$, the conditional distribution is often heteroskedastic or skewed, while ln(*y*) is much less so
- The distribution of ln(*y*) is more narrow, limiting the effect of outliers

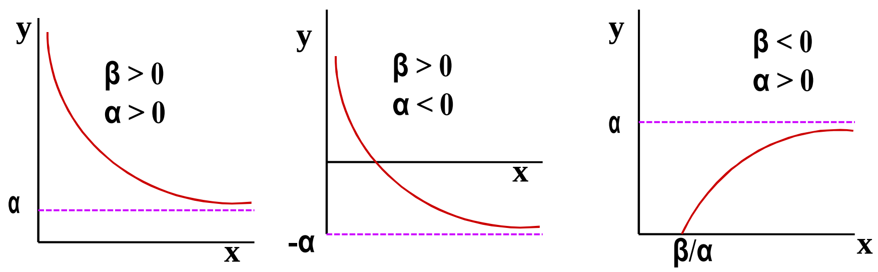
Some Rules of Thumb

- What types of variables are often used in log form?
- Dollar amounts that must be positive
- Very large variables, such as population
- What types of variables are often used in level form?
- Variables measured in years
- Variables that are a proportion or percent

Reciprocal model

$$y_i = \alpha + \beta \frac{1}{x_i} + u_i \qquad \frac{dy}{dx} = -\frac{\beta}{x^2}$$

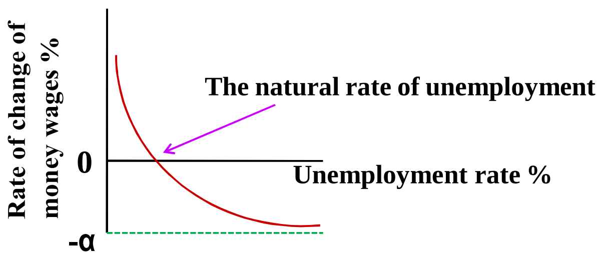
- Slope everywhere negative and decreases in absolute value as x increases.
- As $x \to 0$, $y \to \infty$, and as $x \to \infty$, $y \to \alpha$.



Reciprocal model: Phillips Curve

Percentage rate of change of money wages (y) and unemployment rate of the UK for 1861-1957

AW Phillips (1958) Economica Vol 15: 283-299



Logarithmic reciprocal model

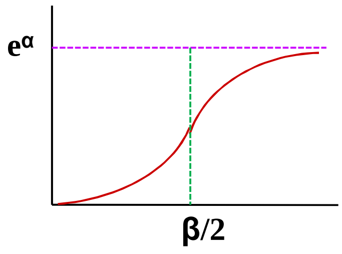
$$\ln y_i = \alpha + \beta \frac{1}{x_i} + u_i \qquad y_i = e^{\alpha - \frac{\beta}{x_i} + u_i}$$

$$\frac{dy}{dx} = \frac{y\beta}{x^2}$$

- Slope is positive for positive x

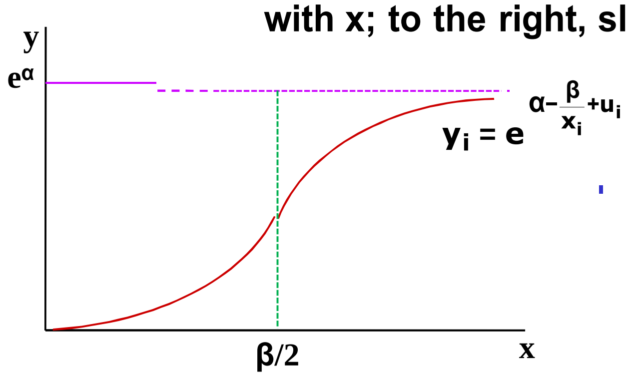
$$\frac{d^2Y}{dX^2} = Y\left(\frac{\beta^2}{X^4} - \frac{2\beta}{X^3}\right)$$

Point of inflexion at x = β/2.



Logarithmic reciprocal model

- Point of inflexion at $x = \beta/2$.
  - To the left of this point, slope ↑ with x; to the right, slope ↓.



- As $x \to \infty$, $y \to e^{\alpha}$

Quadratic Models

- $y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$
- What's the slope here?
- Not β_1 alone

$$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$$

More on Quadratic Models

For $\beta_1 < 0$ and $\beta_2 > 0$,
the turning point is at

$$x^* = \left|\beta_1 / (2\beta_2)\right|,$$

which is the same as when

$\beta_1 > 0$ and $\beta_2 < 0$

Interaction Terms

- $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + u$
- What's the slope wrt x_1 ?
- Not β_1 alone

$$\frac{dy}{dx_1} = \beta_1 + \beta_3 x_2,$$

so to summarize the effect of x_1 on y
we typically evaluate the above at $\bar{x}_2$

Least-squares growth rate.

Least-squares growth rates are used wherever there is a sufficiently long time series to permit a reliable calculation.

The least-squares growth rate, *r,* is estimated by fitting a *linear regression trend line* to the logarithmic annual values of the variable in the relevant period.

The regression equation takes the form

$$\ln y_t = a + bt.$$

The regression equation, $\ln y_t = a + bt$

may be taken as

(i) the logarithmic transformation of the exponential growth equation, $y(t) = y(0)e^{rt}$ **,**

with $a = \ln y(0)$ **and** $b = r$**, the parameters to be estimated.**

If b^* **is the least-squares estimate of *b*, the same gives the average annual exponential growth rate, r,**

and is multiplied by 100 for expression as a percentage.

The regression equation, $\ln y_t = a + bt$

may also be taken as

(ii) the logarithmic transformation of the compound growth equation, $y_t = y_o (1 + r)^t$,

with $a = \ln y_o$ and $b = \ln(1 + r)$.

If b^* is the least-squares estimate of b, the average annual growth rate, r, is obtained as [exp(b^*) – 1].

Remember Compound growth rate =

Exp(exponential growth rate) – 1.

Estimating Growth rate of real state domestic product (SDP) of Kerala for 1960-61 – 1999-2000:

Growth rate of real state domestic product (SDP) of Kerala for 1960-61 – 1999-2000:

```
. regress lnsdp t

  Source |       SS       df       MS              Number of obs =      40
---------+------------------------------           F(  1,    38) =  862.43
   Model |  6.60991811     1  6.60991811           Prob > F      =  0.0000
Residual |  .29124397     38  .007664315           R-squared     =  0.9578
---------+------------------------------           Adj R-squared =  0.9567
   Total |  6.90116208    39  .176952874           Root MSE      =  .08755

------------------------------------------------------------------------------
   lnsdp |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
       t |   .0352155    .0011991     29.37   0.000        .032788    .0376431
   _cons |   13.43204    .0282119    476.11   0.000       13.37492    13.48915
------------------------------------------------------------------------------
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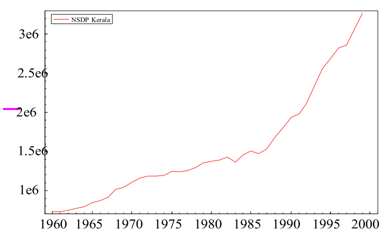
Growth rate : Coefficient of t = 0.035215
= 3.52%

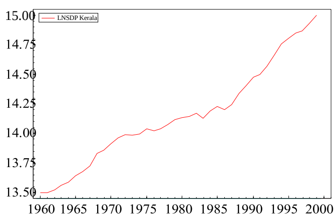
Growth rate of real state domestic product (SDP) of Kerala for 1960-61 – 1999-2000:



ln SDP = 13.432 + 0.0352155 time

(476) (29.4)

R^2 = 0.9578 F(1, 38) = 862.4; n = 40.



Exponential growth rate: 0.0352

or 3.52% per year.

Compound growth rate: exp(0.0352) – 1 = 1.03584 – 1 = 0.03584 or 3.58% per year.